An anti-saturation steering law for Three Dimensional Magnetically Suspended Wheel cluster with angle constraint

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\textbf{ABSTRACT}

Three Dimensional Magnetically Suspended Wheel (3-DMSW) is a new kind of inertia actuator for spacecraft attitude control, which can provide a 3 degrees of freedom torque. On account of the constraint characteristics of 3-DMSW such as a small deflection saturation angle of rotor shaft and the saturation of rotor's variable rotational speed, an anti-saturation steering law based on weighted pseudo inverse is proposed for 3-DMSW cluster. A new weight adjustment method is proposed to adjust the weights of shaft deflections dynamically. A specially designed exponential function with current deflection angle and angular velocity information on the exponent position is adopted as the evaluation criterion of current torque output ability of shaft deflection. Thus the torque command can be distributed dynamically with no angle saturation. The weight adjustment method is demonstrated theoretically and the effectiveness of the anti-saturation steering law is validated by conducting several numerical simulations of attitude agile maneuver. Comparing with the 3-DMSW cluster and flywheel cluster using the traditional steering law, the results show that the 3-DMSW cluster using the proposed method makes the process of agile maneuver more rapid and accurate and the saturation angles of 3-DMSW cluster will not be reached.

\textbf{1. Introduction}

With the miniaturization trend of satellites, more micro-satellites are applied to earth observation such as SkySat-1 (America)\textsuperscript{1} and SuperView-1 (China)\textsuperscript{2}, and they make it possible for commercial space companies to fully expand the business in the space market of the world. Among those different kinds of micro-satellites, earth observation micro-satellites are widely developed because of their practical value. Due to the characteristics of small volume and light weight, earth observation micro-satellites need small and effective attitude control systems to meet the requirements of imaging system payload. The attitude actuator which is of superior performance should be capable of providing instantaneous large control torque for attitude agile maneuver and high-precision control torque for attitude stabilization. It is well-known that Flywheel (FW) is widely used for attitude stabilization. Although FW can also be applied to agile maneuver independently\textsuperscript{3,4}, it seems inefficient because its output torque is usually relatively small. It will reach saturation frequently when attitude maneuver requires a long-term large torque and many other types of actuators are applied to desaturate the rotational speed\textsuperscript{5–7}. In order to meet the requirement of agile maneuver, Control Moment Gyro (CMG) is applied to attitude control. CMG is a kind of attitude actuator which can produce a large torque output with a fairly small torque input, by using a moment wheel with a fixed high rotational speed and gimbals which can adjust its direction of space. Single Gimbal Control Moment Gyro (SGCMG) is widely used in large spacecrafts because of the large torque output, fast response and small mechanical complexity\textsuperscript{8,9}. Meanwhile, Double Gimbal Control Moment Gyro (DGCMG) is applied to attitude control because of the easy description of singularity and multidirectional torque output\textsuperscript{10,11}. Despite the advantages of different kinds of CMGs mentioned above, a high precision control torque output is hard to guarantee and the problem of singularity always exists\textsuperscript{12}. To combine the advantages of FW and CMG, they are integrated in one attitude control system to realize a high-precision attitude stability using FWs and a large torque output using CMGs\textsuperscript{13–16}. However, it makes the attitude control system heavier and more complex which is not suitable for micro-satellites. Ford and Hall first came up with the concept of Variable Speed Control Moment Gyro (VSCMG), which combines the large torque output feature of CMG and the high-precision torque output feature of FW in one actuator\textsuperscript{17}. Double-Gimbal VSCMG (DGVSCMG) is a kind of VSCMG with 2-DOF gimbals, which is capable of generating control torque output in three directions\textsuperscript{18,19}.  

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Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$A_i$</td>
<td>transfer matrix from the $i$th 3-DMSW body coordinate frame to spacecraft body frame</td>
</tr>
<tr>
<td>$B^+$</td>
<td>pseudo inverse of matrix $B$</td>
</tr>
<tr>
<td>$B^\circ$</td>
<td>weighted pseudo inverse of matrix $B$</td>
</tr>
<tr>
<td>$G_i$</td>
<td>3-DMSW rotor body coordinate frame</td>
</tr>
<tr>
<td>$G_f$</td>
<td>3-DMSW body coordinate frame</td>
</tr>
<tr>
<td>$h_f$</td>
<td>total angular momentum of the 3-DMSW cluster expressed in spacecraft body frame, kg-m²/s</td>
</tr>
<tr>
<td>$h_{yi}$</td>
<td>angular momentum of the $i$th 3-DMSW expressed in spacecraft body frame, kg-m²/s</td>
</tr>
<tr>
<td>$I$</td>
<td>axial moment of inertia of the rotor, kg-m²</td>
</tr>
<tr>
<td>$K_{un}$, $K_{\beta_i}$</td>
<td>unsaturation degree of the $i$th rotor shaft deflection angle</td>
</tr>
<tr>
<td>$q$</td>
<td>attitude quaternion of spacecraft</td>
</tr>
<tr>
<td>$R$</td>
<td>attitude actuator instruction vector</td>
</tr>
<tr>
<td>$T_c$</td>
<td>torque command from attitude controller, N-m</td>
</tr>
<tr>
<td>$T_o$</td>
<td>actual torque output for attitude control, N-m</td>
</tr>
<tr>
<td>$W_i$</td>
<td>coefficient of the weight of $i$th shaft deflection angle</td>
</tr>
<tr>
<td>$\omega$</td>
<td>attitude angular velocity of spacecraft, rad/s</td>
</tr>
<tr>
<td>$\omega_{ax}$, $\omega_{ay}$</td>
<td>deflection angular velocity of the $i$th rotor shaft, rad/s</td>
</tr>
<tr>
<td>$\sigma_i$, $\beta_i$</td>
<td>deflection angular velocity of the $i$th 3-DMSW rotor shaft, rad</td>
</tr>
<tr>
<td>$\sigma_i$, $\beta_i$</td>
<td>time derivative of the $i$th 3-DMSW rotor shaft deflection angle, rad/s</td>
</tr>
<tr>
<td>$\sigma_{max}$, $\beta_{max}$</td>
<td>deflection saturation angle of rotor shaft, rad</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>an adjustable parameter of the weight</td>
</tr>
<tr>
<td>$\Omega_f$</td>
<td>initial rotation speed of 3-DMSW cluster, rad/s</td>
</tr>
<tr>
<td>$\Omega_i$</td>
<td>rotation speed of the $i$th rotor, rad/s</td>
</tr>
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</table>

VSCMG, including DGVSCMG, can work in CMG mode when a large torque is needed, and FW mode when a high-precision torque is needed or CMG mode is approaching singularity [20,21]. Although VSCMG has many advantages in satellite attitude control, there are many inherent drawbacks such as friction and output disturbance like other inertial actuators. As is known widely, the rotor of FW, CMG or VSCMG is supported by mechanical bearings which have friction of certain value. With the extension of working hours, the performance of bearings will degrade and the friction in the gimbals will increase which bring about output disturbance [22]. It troubled many spacecrafts in the past few decades.

Due to the advantages of non-contact, non-friction, high precision and long life, Active Magnetic Bearing (AMB) is applied to inertial actuators. With the active vibration control of AMB, the rotor can rotate stably and its vibration can be reduced to infinitely small [23,24]. Three Dimensional Magnetically Suspended Wheel (3-DMSW) is a new kind of inertia attitude actuator based on AMB [25–27]. 3-DMSW has many merits: (1) By applying suitable control methods, 3-DMSW can stably output high-precision radial control torque in 2 degrees of freedom by deflecting the rotor shaft and axial control torque by accelerating and decelerating the rotor [26,27]. (2) 3-DMSW has virtual gimbals and doesn't need real gimbals owing to the application of AMB. As a result, the control of rotor shaft deflection and the control of rotor suspending are highly integrated. (3) 3-DMSW can output a fast response control torque with a high band-width. (4) This innovative actuator can measure the angular rates of spacecraft in 2 degrees of freedom which makes it a good attitude sensor simultaneously [25]. All merits mentioned above create a good application prospect for 3-DMSW in the near future.

In order to apply 3-DMSW cluster to the attitude control system, a steering law is needed to distribute the torque command properly. The traditional pseudo inverse steering law is proposed to meet the requirement of a minimum power consumption [13]. However, it is not suitable for 3-DMSW because the deflection angle of 3-DMSW rotor shaft is limited [28]. Some steering laws are proposed to avoid singularity at the cost of allowing torque output error. Singularity robust inverse steering law and singular direction avoidance steering law are representative among them [10,29–31]. All of them are proposed to solve the singularity problem of CMG cluster while this problem is not involved in 3-DMSW. Weighted pseudo inverse steering law is successfully applied to VSCMG cluster to avoid singularity. In this kind of steering law, the weight of FW mode can be adjusted dynamically according to the current singularity indicator of the gimbals [20,21,32]. However, it can not be applied to 3-DMSW directly because the problem of small deflection saturation angle of 3-DMSW is not involved in the steering law for VSCMG cluster. So in this paper, an anti-saturation steering law based on weighted pseudo inverse method is proposed. In this steering law, the weight of CMG mode is constructed in a special form of an exponential function in which the unsaturation degree of rotor deflection angle is included. By using this weight adjustment method, the weight of CMG mode can vary dynamically according to the current deflection angle and deflection angular velocity. As a result, the deflection angle of rotor shaft will not reach saturation. This innovative work makes it possible to apply 3-DMSW cluster to the real attitude control system of earth observation micro-satellites.

The remainder of the paper is briefly outlined as follows. In Sec.2, the coordinate frame definition of 3-DMSW is given and the output torque produced by 3-DMSW is presented clearly. In Sec.3, the new anti-saturation weighted pseudo inverse steering law is proposed and the specific formulas of the weight adjustment method are presented to meet the angle constraint of 3-DMSW. In Sec.4, numerical simulations are performed to validate the effectiveness of the anti-saturation steering law which contains a new weight adjustment method specially designed for 3-DMSW cluster, by comparing with the traditional pseudo inverse steering law and the case using flywheels only. Finally, the conclusions are drawn in Sec.5.

2. Basic formulation of 3-DMSW cluster

3-DMSW is mainly composed of axial/deflection magnetic bearings, radial magnetic bearings, displacement sensor, motor, rotor, shaft, base and seal cover if classified by functions which is shown in Fig. 1 [25]. For the convenience of studying the dynamic characteristics, it can be simplified to a machine having a rotor suspended by magnetic bearings as shown in Fig. 2. The non-contact displacement sensor measures the translational motion and the rotational motion of rotor. In order to describe the angular momentum of the 3-DMSW, the coordinate frames shown in Fig. 3 are defined as follows.

1) Body coordinate frame of 3-DMSW $G_f$ (Ox$_{ef}$y$_{ef}$z$_{ef}$): The $G_f$ frame is the orientation of the entire 3-DMSW. The origin is located at the mass center of the rotor. The $z_f$ axis lies along the initial symmetrical axis...
of the rotor shaft with no deflection. The \( x_f \) axis and \( y_f \) axis are parallel to the \( x \) axis and \( y \) axis of AMB and perpendicular to the \( z_f \) axis. They follow the right-hand law.

2) Body coordinate frame of 3-DMSW rotor \( G_b \) (\( O_{x_b}, y_b, z_b \)): The \( G_b \) frame is fixed on the rotor but does not rotate with it. Its origin is located at the mass center of the rotor. The rotor rotates about the \( z_b \) axis with an angular velocity \( \Omega \). The \( x_b \) axis and \( y_b \) axis are parallel to the \( x_f \) axis and \( y_f \) axis in the initial state with no deflection and follow the right-hand law.

Here the basic coordinates transfer sequence is defined as follows: First rotate \( G_f \) frame along \( x_f \) axis with an \( \alpha \) angle, and then rotate \( G_f \) frame along \( y_f \) axis with a \( \beta \) angle. Thus \( G_f \) frame is reached. \( \dot{\alpha} \) and \( \dot{\beta} \) are rotational angular velocities along \( x_b \) axis and \( y_b \) axis respectively.

Before the derivation of the basic formulation, some assumptions should be clarified: 1) 3-DMSW is a type of momentum wheel whose maximum deflection torque is larger than its rotary torque. 2) The minimum quantity of 3-DMSW in the cluster is 3 in order to produce continuous effective torque output in any direction in space. 3) Only polar moment of inertia of 3-DMSW's rotor is considered in the basic formulation.

Based on the definition of the coordinate frame above, the total angular momentum of the 3-DMSW cluster which is composed of at least three 3-DMSWs can be expressed in the spacecraft body frame as

\[
h_i = \sum_{i=1}^{n} A_i/\hat{h}_i = A/\hat{h}_f
\]

where \( \hat{h}_i \) is the angular momentum of each 3-DMSW expressed in the spacecraft body frame. \( n \) is the total number of 3-DMSW used in the cluster. \( I \) is the axial moment of inertia of the rotor. \( A_i \) presents the transfer matrix from the body frame of each 3-DMSW to spacecraft body frame and \( A = [A_1, A_2, ..., A_n] \). \( \hat{h}_i \) is defined as follows

\[
\hat{h}_i = \Omega_i \begin{bmatrix} \sin \beta_i \\ \cos \alpha_i \cos \beta_i \end{bmatrix}
\]

(2)

where \( \alpha_i \) and \( \beta_i \) are the current deflection angles of rotor shafts. \( \Omega_i \) is the angular velocity of rotor. \( \hat{h}_i = [\hat{h}_{i1}, \hat{h}_{i2}, ..., \hat{h}_{in}] \).

According to the theorem of angular momentum, the output torque of 3-DMSW cluster can be given by the time derivative of \( \hat{h}_f \) as follows

\[
T_e = -\hat{h}_f = -A\hat{h}_f = BR
\]

(3)

where \( B = [B_1, B_2, ..., B_n] \), \( R = [R_1^T, R_2^T, ..., R_n^T]^T \), and

\[
B_i = A_i \begin{bmatrix} 0 & -\Omega_i \cos \beta_i & -\sin \beta_i \\ \Omega_i \cos \alpha_i \cos \beta_i - \Omega_i \sin \alpha_i \sin \beta_i & -\sin \alpha_i \cos \beta_i & \sin \alpha_i \cos \beta_i \\ \Omega_i \sin \alpha_i \cos \beta_i & \Omega_i \cos \alpha_i \sin \beta_i & -\cos \alpha_i \cos \beta_i \end{bmatrix}
\]

(4)

\[
R_i = [\delta_i \quad \beta_i \quad \Omega_i]^T
\]

(5)

Here the output torque produced by 3-DMSW cluster has been presented. When a specific \( R \) is presented, an output torque \( T_e \) will be generated. Only if the deflection angle \( \alpha \) and \( \beta \) do not reach the maximum value, \( \hat{\alpha} \) and \( \hat{\beta} \) will not be forced to zero and the output torque will be generated correctly. Due to the deflection angle constraint of 3-DMSW, a new steering law should be proposed to obtain \( R \) and to avoid angle saturation.

3. Anti-saturation steering law with new weight adjustment method

The closed-loop attitude control system for the micro-satellite equipped with 3-DMSW cluster is shown in Fig. 4. It can be seen that the steering law should obtain the actuator instruction \( R \), which includes the angular velocity of the rotor shaft deflection and the rotational acceleration of the rotor. So that the actual output torque \( T_e \)
produced by 3-DMSW cluster can equal to the command torque $T_c$ as far as possible.

Traditional steering law for CMG or FW cluster uses pseudo inverse method to obtain $R$ which has a minimum power consumption, then $R$ is given by

$$R = B^*T_c$$  \hspace{1cm} (6)

where $B^*$ is the pseudo inverse of matrix $B$ as is follows

$$B^* = B^T(BB^T)^{-1}$$  \hspace{1cm} (7)

However, for 3-DMSW cluster, pseudo inverse steering law can not distribute the torque command properly, because the elements of matrix $B$ which represent CMG part and FW part differ greatly in order of magnitude due to the presence of $\Omega_i$ in CMG part. In order to distribute the torque command properly, weighted pseudo inverse method is adopted in this anti-saturation steering law. Here $R$ is given as

$$R = B^*T_c$$  \hspace{1cm} (8)

where $B^*$, the weighted pseudo inverse matrix of $B$ is as follows

$$B^* = WB^T(BWB^T)^{-1}$$  \hspace{1cm} (9)

In Eq. (9), $W$ is a diagonal matrix which takes the form of

$$W = \text{diag}(W_{1w}, W_{2w}, W_{3w}, W_{4w}, W_{5w}, W_{6w}, W_{7w}, W_{8w})$$  \hspace{1cm} (10)

in which $W_{1w}$ and $W_{2w}$ represent the weights of rotor shaft's deflections in two directions and $W_{3w}$ represents the weight of rotor's acceleration.

In order to adjust the torque distribution logic dynamically, the weight mentioned above should be adjusted in real time according to the current status. For spacecraft agile maneuver, an instant large torque output is always required. So when a large angle attitude maneuver command comes, the CMG mode should take the dominant position immediately to provide a large torque output until the target angle is approached or the deflection angle of rotor shaft is near saturated. When the deflection angle is near saturated and an output torque is still needed, the FW mode should take over the duty of the CMG mode to output control torque continuously. Based on the steering logic mentioned above, an anti-saturation weight adjustment method is proposed as follows.

For the rotor shaft deflection, the weight is set in a specially designed form of exponential function $y = e^{x\alpha}$ as follows

$$W_{ii} = W_0e^{\frac{\alpha_i}{\beta_i}x_{ii}^{\frac{1}{\alpha_i}}}, \quad W_{ij} = W_0e^{\frac{\alpha_i}{\beta_j}x_{ij}^{\frac{1}{\alpha_i}}}$$  \hspace{1cm} (11)

where $W_0$ is the coefficient of weight which is set as 1, $\alpha_i$ and $\beta_i$ are deflection angular velocities of the $i$th rotor shaft, $\frac{\alpha_i}{\beta_i}$ and $\frac{\alpha_i}{\beta_j}$ represent the sign of the current $\omega_i$ and $\omega_j$ respectively. Their product can effect the current sign of exponent $\alpha$ which is a parameter that can be adjusted manually. $K_{\alpha i}$ and $K_{\beta i}$ which represent the unsaturation degrees of the deflection angles are set as

$$K_{\alpha i} = |\alpha_i| = \alpha_{\max i}$$  \hspace{1cm} (12)

where $\alpha_{\max i}$ and $\beta_{\max i}$ are the maximum deflection angles of rotor shafts.

For the rotor acceleration, the weight is set simply as

$$W_{k} = 1$$  \hspace{1cm} (13)

The exponential function $y = e^{\frac{1}{x}}$ which is adopted in Eq. (11) can adjust the weight of CMG mode flexibly with $xK_{\alpha i}$ or $xK_{\beta i}$ at $x$ position in order to avoid deflection saturation. Combining with the function image shown in Fig. 5, the anti-saturation effect of the new weight adjustment method can be demonstrated as follows.

There are three typical states according to the signs of the current deflection angle and angular velocity:

1) When the deflection angle is very small and far from saturation, the unsaturation degree $K_{\alpha i}$ or $K_{\beta i}$ is relatively large. In this state, no matter what the sign of angular velocity is, the exponential function $y = e^{\frac{1}{x}}$ or $y = e^{-\frac{1}{x}}$ will make the weight close to 1 which is large enough to make 3-DMSW work in CMG mode.

2) When the deflection angle is near saturation, the unsaturation degree $K_{\alpha i}$ or $K_{\beta i}$ is relatively small. In this state, if the sign of angular velocity is the same as the sign of deflection angle, the exponential function $y = e^{\frac{1}{x}}$ will make the weight close to 0 to stop the deflection. Meanwhile the FW mode will take over the torque output responsibility.

3) When the unsaturation degree is relatively small but the sign of angular velocity is opposite to the sign of deflection angle, the exponential function $y = e^{-\frac{1}{x}}$ will make the weight close to positive infinity to enhance the deflection.

So far, the total anti-saturation steering law which uses the new weight adjustment method has been formulated completely.

4. Simulations

To demonstrate the effectiveness of the anti-saturation steering law for 3-DMSW cluster using new weight adjustment method, numerical simulations are performed. The target of the attitude control is to realize an attitude maneuver of $10^\circ$ rotation about the roll axis of spacecraft. A proportional-derivative (PD) controller is adopted and the parameters of PD controller are $K_p = 0.3$, $K_d = 0.8$. The 3-DMSW cluster used in the simulation has a three orthogonal configuration which is illustrated in Fig. 6. The installation direction of the cluster is defined as $z_F$ axis, the first 3-DMSW is along roll axis. The $z_F$ axis of the second 3-DMSW is along pitch axis. The $z_F$ axis of the third 3-DMSW is along yaw axis. Roll, pitch and yaw axes are corresponding to $x$, $y$ and $z$ axes of spacecraft body frame.

The simulations are done in three cases. Case 1 uses the anti-saturation steering law with new weight adjustment method represented in Eq. (8)-Eq. (13). Case 2 uses the traditional pseudo inverse steering law represented in Eq. (6)-Eq. (7). Case 3 uses flywheels only with pseudo inverse steering law. The model parameters of the attitude control system are listed in Table 1. Then the performances of attitude control, the accuracy and tracking performance of torque output are presented. The variations of deflection angles and rotational speeds of the rotors are presented in the end.

![Fig. 5. The exponential function adopted in the weight adjustment method.](image-url)
The figure of spacecraft attitude angle varying process during the spacecraft agile maneuver is described in Fig. 7. It can be seen that when the anti-saturation steering law with new weight adjustment method is applied to 3-DMSW cluster, the roll angle reaches the target value rapidly while the pitch angle and yaw angle both have a slight fluctuation and finally return to zero. Meanwhile, when the traditional pseudo inverse steering law is applied to 3-DMSW cluster, the roll angle reaches the target value stably. As for the flywheel cluster using pseudo inverse steering law, it takes a much longer time to realize the agile maneuver.

The contrast figures between torque command and actual torque output, which represent the output accuracy and tracking performance of the actuator, are described in Figs. 8–10. In these figures, $T_{cc}$ and $T_{ct}$ represent the torque command from controller and the actual torque output along $x$ axis of the spacecraft body frame respectively. Similarly, $T_{cy}$ and $T_{ct}$ are along $y$, $T_{cx}$ and $T_{ct}$ are along $z$ axis. It can be seen from Fig. 8 that the 3-DMSW cluster using the anti-saturation steering law with new weight adjustment method has a good performance of tracking the torque command coming from the attitude controller in the early stage of the attitude maneuver. On account of the electric motor’s capability of acceleration is limited, when 3-DMSW working in FW mode, there is a maximum torque output limitation which is set to 2.166rad/s$^2$ in the numerical simulation. Owing to the maximum torque output limitation of FW mode, the figure shows a decrease in output torque magnitude in the later stage though, it is enough for the spacecraft to accomplish the attitude maneuver task. In contrast, the performance of 3-DMSW cluster using the traditional pseudo inverse steering law is shown in Fig. 9. It can be seen that the torque command and output both have intense oscillation and can not meet the requirement of the attitude maneuver. Meanwhile, the performance of using flywheel cluster is shown in Fig. 10. Similar with the 3-DMSW cluster in FW mode, the maximum torque output limitation always exists when using flywheel cluster only, which has a significant negative impact on the attitude maneuver process.

The figures of rotor shafts’ deflection angles and rotational speeds are shown in Figs. 11–13. It can be seen from Fig. 11 that due to the application of the new weight adjustment method, all the deflection angles range within the limit of deflection saturation angle and 3-DMSW cluster works in FW mode properly in the middle and later stages of agile maneuver. It is worth noting that when the attitude maneuver is finished after 50s, the deflection angles and rotational speeds are still changing slowly and produce small torque output on account of the existence of spacecraft orbit angular velocity. Fig. 12 shows that the deflection angles are limited by the deflection saturation angle of rotor shaft itself. The rotational speeds of 3-DMSW cluster don’t change obviously which illustrates that the traditional pseudo inverse method can not distribute command torque to FW mode properly and makes 3-DMSW cluster weak in attitude control. Fig. 13 shows that if using flywheel cluster only, the rotational speeds of flywheels vary in a much larger range than using 3-DMSW cluster. This will bring about much more pressure for the desaturation of flywheels.

5. Conclusions

Three Dimensional Magnetically suspended wheel (3-DMSW) is a new kind of suitable inertia actuator for earth observation micro-satellites, because it is able to provide either instant large torque for attitude agile maneuver or high precision torque for attitude stabilization with smaller mass and volume. An anti-saturation steering law for 3-DMSW cluster works in FW mode properly in the middle and later stages of agile maneuver. It is worth noting that when the attitude maneuver is finished after 50s, the deflection angles and rotational speeds are still changing slowly and produce small torque output on account of the existence of spacecraft orbit angular velocity. Fig. 12 shows that the deflection angles are limited by the deflection saturation angle of rotor shaft itself. The rotational speeds of 3-DMSW cluster don’t change obviously which illustrates that the traditional pseudo inverse method can not distribute command torque to FW mode properly and makes 3-DMSW cluster weak in attitude control. Fig. 13 shows that if using flywheel cluster only, the rotational speeds of flywheels vary in a much larger range than using 3-DMSW cluster. This will bring about much more pressure for the desaturation of flywheels.

### Table 1
Parameters of the attitude control system.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I$</td>
<td>Polar inertia of 3-DMSW rotor</td>
<td>0.0277kg m$^2$</td>
</tr>
<tr>
<td>$J$</td>
<td>Moment of inertia of satellite</td>
<td>8.83 0.68 − 0.16</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.68 7.70 − 0.10</td>
</tr>
<tr>
<td></td>
<td></td>
<td>−0.16 − 0.10 15.0</td>
</tr>
<tr>
<td>$\Omega_0$</td>
<td>Initial rotation speed of 3-DMSW rotor</td>
<td>2500rpm</td>
</tr>
<tr>
<td>$\Omega_{\text{max}}$</td>
<td>Maximum rotation speed of 3-DMSW</td>
<td>5000rpm</td>
</tr>
<tr>
<td>$\Omega_{\text{max}}$</td>
<td>Maximum acceleration of the rotation speed of 3-DMSW</td>
<td>2.166rad/s$^2$</td>
</tr>
<tr>
<td>$\beta_{0}$</td>
<td>Initial deflection angle of rotor shaft</td>
<td>0°</td>
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<tr>
<td>$\gamma_{\text{max}}$, $\beta_{\text{max}}$</td>
<td>Deflection saturation angle of rotor shaft</td>
<td>3°</td>
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<tr>
<td>$\omega_{\text{orb}}$</td>
<td>Spacecraft orbit angular velocity</td>
<td>0.0001rad/s</td>
</tr>
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</table>

Fig. 6. Configuration of 3-DMSW cluster.

Fig. 7. Response of attitude angles during spacecraft agile maneuver.

Fig. 8. Response of roll angle.

Fig. 9. Response of pitch angle.

Fig. 10. Response of yaw angle.

Fig. 11. Comparison of deflection angles and rotational speeds for 3-DMSW cluster using anti-saturation steering law with new weight adjustment method and traditional pseudo inverse steering law.

Fig. 12. Comparison of deflection angles and rotational speeds for 3-DMSW cluster using traditional pseudo inverse steering law and flywheels cluster.

Fig. 13. Comparison of deflection angles and rotational speeds for 3-DMSW cluster using flywheels cluster only.
Fig. 8. Torque command and output of 3-DMSW cluster using anti-saturation steering law.

(a) Roll axis  (b) Pitch axis  (c) Yaw axis

Fig. 9. Torque command and output of 3-DMSW cluster using pseudo inverse steering law.

(a) Roll axis  (b) Pitch axis  (c) Yaw axis

Fig. 10. Torque command and output of flywheel cluster using pseudo inverse steering law.

(a) Roll axis  (b) Pitch axis  (c) Yaw axis

Fig. 11. Rotor shaft deflection angles and rotational speeds of 3-DMSW cluster using anti-saturation steering law.

(a) Deflection angle $\alpha$  (b) Deflection angle $\beta$  (c) Rotational speed

Fig. 11. Rotor shaft deflection angles and rotational speeds of 3-DMSW cluster using anti-saturation steering law.
weights of rotor shafts’ deflection saturation angles are not reached and the rotational speeds of the rotors vary within a smaller range.

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