LOW-EARTH ORBIT DETERMINATION BASED ON ATMOSPHERIC DRAG MEASUREMENTS

Rui Zhang, Fei Xu, Chao Han, and Xiucong Sun

An autonomous low-Earth orbit determination method based on measurements of atmospheric drag is proposed, with use of a high-precision tri-axis accelerometer as well as a star sensor. A set of equations is formulated to determine current position and velocity of the spacecraft using drag measurements of at least four epochs. Magnitude of the drag acceleration vectors is not used in the proposed method in order to avoid orbit determination error due to the uncertainty of the Earth atmosphere model. Analytical initial values of position and velocity which is used to solve the orbit determination equations numerically is provided under the assumption of circular orbit. Numerical simulations are performed which show good performance of the novel orbit determination method. This method is useful for the navigation of low-Earth-orbiting spacecraft in the environments where there is no global navigation satellite system.

INTRODUCTION

Geophysical information, such as Earth atmosphere, gravity or magnetic field, is of growing interest and concern for the use of fully autonomous or GPS-denied navigation.1 Gravity gradients measurements which are provided by spaceborne gravity gradiometer have been used to obtain position by matching the observations with a precise gravity model. With use of Extended Kalman Filter (EKF), measurement noise has been reduced through incorporation of the orbital motion and radial/cross-track position errors of less than 100m have been achieved with real flight data of gravity field and steady-state ocean circulation explorer (GOCE) satellite.2,3,4,5 Spaceborne magnetometers which provide magnetic field measurements have been proposed for autonomous spacecraft orbit and attitude determination. By matching the observations with the International Geomagnetic Reference Field (IGRF) model, position errors of several kilometers and attitude errors from 0.1 to 5 deg have been achieved with simulated and real flight data.6,7,8,9

Accelerometer, which is able to measure the non-conservative force acceleration including atmospheric drag acceleration, has been used to aid Inertial Navigation System (INS). Over the past few decades, the measurement accuracy of the accelerometer is getting higher. For example, the GRADIO electrostatic accelerometer which is developed by ONERA has reached a noise density level of $10^{-12} \text{m/s}^2/\sqrt{\text{Hz}}$ which is accurate enough for the measurement of atmospheric drag.10,11 A combined navigation method which uses Drag Derived Altitude (DDA) aided inertial navigation system has been proposed for the navigation of reentry vehicle in the blackout zone. Using DDA can realize a higher navigation accuracy than pure classic inertial navigation algorithm.12,13,14

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For low-Earth orbiting (LEO) spacecraft, atmospheric drag can be measured by a spaceborne tri-axis accelerometer continuously. So in this paper, an autonomous low-Earth orbit determination method is proposed based on the atmospheric drag measurements of an on-board tri-axis accelerometer. Considering the aerodynamic drag formula about the spacecraft velocity relative to the atmosphere, density of atmosphere and other parameters, a set of equations is derived to determine the current position and velocity of the spacecraft. In order to avoid the orbit determination error due to the uncertainty of the Earth atmosphere model, the magnitude of drag acceleration vector is not used in the proposed method. Analytical initial values of position and velocity which is used to solve the orbit determination equations numerically is provided under the assumption of circular orbit. With the help of a star sensor, the coordinate-transformation matrix from spacecraft body coordinate system to Earth Centered Inertial (ECI) coordinate system can be obtained. Then the expression of the accelerometer measurements under the spacecraft body coordinate system can be transformed to ECI coordinate system.

The remainder of the paper is briefly outlined as follows: In Section 2, the basic information of the earth atmosphere is introduced. In Section 3, the orbit determination equations are formulated in detail. The preprocessing of accelerometer measurements is introduced and the initial value formulas are derived. In Section 4, numerical simulations are conducted. The effect of measurement noise on the accuracy of orbit determination result is analyzed. The proposed method is validated by applying it to a simulated real orbit and the range of application is clarified by a series of simulations. Conclusions are drawn in Section 5.

**BASIC INFORMATION OF THE EARTH ATMOSPHERE**

Atmospheric drag is continuously applied to the LEO spacecraft and can be measured by accelerometer. The magnitude of the drag acceleration is shown in Figure 1 which is acquired from a series of orbit with altitudes from 200km to 2000km. The area-mass ratio is set to a common value of 0.01 and SA76 density model of atmosphere is adopted. Figure 1 shows that when orbit altitude is 2000km, the order of magnitude of drag acceleration reaches the minimum $10^{-10} \text{m/s}^2$ which can still be measured by the most advanced accelerometer nowadays. This makes it possible to apply acceleration measurements to the autonomous navigation of LEO spacecraft. As is known widely, there is a relationship between the atmospheric drag and the velocity of spacecraft, which constructs the bridge between atmospheric drag acceleration measurements and orbit parameters. That is to say, if a series of acceleration measurements is obtained, the position and velocity of spacecraft will be determined by conducting some calculations.

**ORBIT DETERMINATION ALGORITHM**

**Definitions of coordinate systems**

In order to present the novel orbit determination algorithm clearly, the coordinate systems used in this paper are defined as follows:

1) Earth Centered Inertial(ECI) frame $G_i(Ox_iy_iz_i)$: The origin of ECI frame is located at the center of the earth. $x_i$ axis is in the earth’s equatorial plane and along vernal equinox direction. $z_i$ axis is perpendicular to the equatorial plane and points the North pole. $y_i$ axis is in the equatorial plane, determined by right-hand rule.

2) Earth Centered Earth Fixed(ECEF) frame $G_f(Ox_fy_fz_f)$: The origin of ECEF frame is located at the center of the earth. The $x_f$ axis is in the earth’s equatorial plane and pointing to the Greenwich
3) Vehicle Velocity Local Horizontal(VVLH) frame $G_o(Ox_0y_0z_0)$: The origin of VVLH frame is located at the mass center of the spacecraft. $z_o$ axis is aligned with opposite of spacecraft’s position. $y_o$ axis is aligned with opposite of orbital momentum. $x_o$ points forward, determined by right-hand rule.

4) Spacecraft body frame $G_b(Ox_by_bz_b)$: The origin is located at the mass center of spacecraft. The frame is fixed with the spacecraft and defined in the process of design. For three-axial-stabilized spacecraft, the body frame is aligned with VVLH frame when there is no attitude error or attitude maneuver.

5) Tri-axis accelerometer body frame $G_t(Ox_ty_tz_t)$: The origin is located at the mass center of the accelerometer. $x_t$, $y_t$, $z_t$ is aligned with three directions of measurement which are perpendicular to each other.

In order to simplify the calculation, the tri-axis body frame is set to coincide with the spacecraft body frame. So the acceleration vector measured by tri-axis accelerometer can be expressed under spacecraft body frame without a step of coordinate transformation.

**Basic equations for orbit determination**

Drag formula is as follows

$$a_{di} = -\frac{1}{2}C_d \frac{S}{m} \rho v_a v_a$$

In this formula, $a_{di}$ is the drag acceleration vector. $C_d$ is the drag coefficient. $\frac{S}{m}$ is the area-mass ratio which is determined by the overall design of the spacecraft and the windward direction. $\rho$ is the local density of the atmosphere. $v_a$ is the spacecraft velocity relative to the atmosphere and $v_a$...
is the magnitude. Considering the earth rotational velocity, the velocity relative to the atmosphere can be expressed as \( \mathbf{v}_a = \mathbf{v} - \mathbf{\omega}_E \times \mathbf{r} \). So the negative normalized drag acceleration vector \( \mathbf{u}_{di} \) can be written as

\[
\mathbf{u}_{di} = \frac{-\mathbf{a}_{di}}{|\mathbf{a}_{di}|} = \frac{\mathbf{v} - \mathbf{\omega}_E \times \mathbf{r}}{|\mathbf{v} - \mathbf{\omega}_E \times \mathbf{r}|}
\]

Then the spacecraft velocity relative to ECI frame can be derived as follows

\[
\mathbf{v} = \mathbf{u}_{di} \cdot |\mathbf{v} - \mathbf{\omega}_E \times \mathbf{r}| + \mathbf{\omega}_E \times \mathbf{r}
\]

and the velocity unit vector can be written as follows

\[
\mathbf{u}_{vi} = \frac{\mathbf{v}}{|\mathbf{v}|} = \frac{\mathbf{u}_{di} \cdot |\mathbf{v} - \mathbf{\omega}_E \times \mathbf{r}| + \mathbf{\omega}_E \times \mathbf{r}}{|\mathbf{u}_{di} \cdot |\mathbf{v} - \mathbf{\omega}_E \times \mathbf{r}| + \mathbf{\omega}_E \times \mathbf{r}|}
\]

Then the orbit determination equations can be written as follows

\[
\begin{cases}
\mathbf{u}_{vi1} = \frac{\mathbf{v}_1}{|\mathbf{v}_1|} \\
\mathbf{u}_{vi2} = \frac{\mathbf{v}_2}{|\mathbf{v}_2|} \\
\mathbf{u}_{vi3} = \frac{\mathbf{v}_3}{|\mathbf{v}_3|} \\
\vdots \\
\mathbf{u}_{vin} = \frac{\mathbf{v}_n}{|\mathbf{v}_n|}
\end{cases}
\]

in which \( \mathbf{u}_{vin} \) is the unit vector of velocity . \( \mathbf{v}_n \) can be obtained by an analytical orbit predictor \( P \) as follows

\[
\mathbf{v}_n = P (\mathbf{r}_1, \mathbf{v}_1, (n - 1) \Delta t)
\]

Let

\[
\mathbf{u}_v = \begin{bmatrix}
\mathbf{u}_{vi1} \\
\mathbf{u}_{vi2} \\
\mathbf{u}_{vi3} \\
\vdots \\
\mathbf{u}_{vin}
\end{bmatrix}
\]

then the orbit determination equations can be rewritten as follows

\[
\mathbf{u}_v = H (\mathbf{r}_1, \mathbf{v}_1, \Delta t, n)
\]

in which \( H \) is function of \( \mathbf{r}_1, \mathbf{v}_1, \Delta t, n \).

**Preprocessing of accelerometer measurements**

It worth noting that all non-conservative force can be measured by accelerometer including atmospheric drag, solar radiation pressure, thrust of the rocket engine and so on. Hence when using the orbit determination method proposed in this paper, the solar radiation pressure acceleration should be removed from the measurements of accelerometer and the rocket engine should be turn off.

By the measurement of tri-axis accelerometer, the non-conservative force acceleration can be expressed under spacecraft body frame as follows

\[
\mathbf{a}_b = \begin{bmatrix}
a_{bx} \\
a_{by} \\
a_{bz}
\end{bmatrix}
\]
in which \(a_{bx}, a_{by}\) and \(a_{bz}\) are \(x, y\) and \(z\) component of the non-conservative force acceleration vector respectively under spacecraft body frame. After the first time measurement, measure every other time interval \(\Delta t\) so that several non-conservative force acceleration measurements \(\{a_{b1}, a_{b2}, \ldots, a_{bn}\}\) can be obtained. In order to keep the observability of the orbit determination method, \(n \geq 4\) should be guaranteed.

The coordinate transformation matrix \(L_{ib}\) from spacecraft body frame to ECI frame can be acquired by star sensor. So the non-conservative force acceleration vectors can be expressed under ECI frame as follows

\[
a_i = L_{ib} \cdot a_b = \begin{bmatrix} a_{ix} \\ a_{iy} \\ a_{iz} \end{bmatrix}
\]

in which \(a_{ix}, a_{iy}\) and \(a_{iz}\) are \(x, y\) and \(z\) component of the non-conservative force acceleration vector respectively under ECI frame. Then all of the non-conservative force acceleration measurements can be transformed to ECI frame as \(\{a_{i1}, a_{i2}, \ldots, a_{in}\}\).

A main disturbance within the measurements of accelerometer is solar radiation pressure acceleration. Here it is removed as follows

\[
a_{di} = a_i - a_{si}
\]

in which \(a_{si}\) can obtained using the ephemeris of the Sun, earth and Moon, current time, current position of spacecraft and other parameters of spacecraft. Then measurements of drag acceleration is \(\{a_{d1}, a_{d2}, \ldots, a_{dn}\}\).

On account of the large uncertainty of earth atmosphere model, the drag acceleration vectors are normalized and taken the opposite direction as follows so that only direction information is used in the proposed method

\[
u_{di} = -\frac{a_{di}}{|a_{di}|}
\]

Then \(\{\nu_{d1}, \nu_{d2}, \ldots, \nu_{dn}\}\) can be obtained which represent approximate spacecraft velocity unit vectors. Noting that due to the rotational effect of the atmosphere along with the earth, a modification of \(\{\nu_{d1}, \nu_{d2}, \ldots, \nu_{dn}\}\) should be made in order to obtain the real velocity unit vector which can be used in the orbit determination equations proposed above.

**Initial value formulas**

In order to solve Eq.(8) numerically, a set of initial value formulas are derived under the assumption of circular orbit. Firstly, normalize the first and the second non-conservative force acceleration as follows

\[
\begin{align*}
u_{i1} &= -\frac{a_{i1}}{|a_{i1}|} \\
u_{i2} &= -\frac{a_{i2}}{|a_{i2}|}
\end{align*}
\]

Then the difference between true anomalies is

\[
\Delta \theta = \arccos (\nu_{i1} \cdot \nu_{i2})
\]

Then the semi-major axis is

\[
a = \left(\frac{\mu \Delta t^2}{\Delta \theta^2}\right)^{\frac{3}{2}}
\]
in which \( \mu = GM \) is the earth gravitational constant. \( G \) is gravitational constant and \( M \) is mass of the earth.

And the magnitude of spacecraft orbit velocity is

\[
\nu_{\text{twobody}} = \sqrt{\frac{\mu}{a}}
\]  

then the initial value of the velocity vector can be obtained as follows

\[
\mathbf{v}_1 = \nu_{\text{twobody}} \mathbf{u}_i_1
\]  

Next, a direction vector of orbit angular momentum can be calculated as follows

\[
\mathbf{\hat{H}} = \mathbf{u}_i_1 \times \mathbf{u}_i_2
\]

then the initial value of the position vector can be obtained as follows

\[
\mathbf{r}_1 = \frac{\mathbf{u}_i_1 \times \mathbf{\hat{H}}}{|\mathbf{u}_i_1 \times \mathbf{\hat{H}}|} \cdot a
\]

Finally, the initial values \( \{ \mathbf{r}_1, \mathbf{v}_1 \} \) for solving the orbit determination equations is presented. Numerical iteration algorithms, like Newton Method, can be adopted to solve the equations because the initial values are very close to the real values. This set of initial values are also needed in the first step calculation of Eq. (4) and Eq. (11).

A flowchart which presents the whole process of the proposed orbit determination method is shown in Figure 2.

So far, the low-earth orbit determination method based on atmospheric drag measurements has been formulated completely.

**SIMULATION**

**Accuracy of the orbit determination method**

To analyze the effect of accelerometer measurement noise and star sensor measurement noise on the orbit determination result, a series of calculations is performed. Keplerian orbits are simulated which consider center gravity of the earth only in order to guarantee the accuracy of calculation. Mean position errors due to white noises of accelerometer and white noises of star sensor are listed in Tables 1 and 2.

Table 1 shows that the position accuracy improves with the decrease of drag acceleration measurement error. As the standard derivation of error decreases for one order of magnitude, the position accuracy increases for one order of magnitude. Moreover, Table 1 shows that the position accuracy gets worse with the increase of altitude. When the altitude and the standard derivation of error is too high, no result can be obtained using the method proposed. As is known widely, the atmosphere density decreases rapidly when altitude is in the range of 200–600km. The standard derivation of measurement error like \( 10^{-8} \text{m/s}^2 \) is quite large and it will strongly affect the drag acceleration measurements in a higher orbit which makes this problem not being solved.
Begin
Non-conservative force acceleration measurement $a_i$

Get initial values of position and velocity $r_i, v_i$

Remove solar radiation pressure acceleration $a_s = a_i - a_d$

Normalization of drag acceleration $u_\delta$

Get normalized velocities $u_v$

Establish the orbit determination equations $u_v = H(r, v, \Delta t, n)$

Solve the equations with numerical methods

Converge?

Y

End

N

Figure 2. Flowchart of the low-Earth orbit determination method based on atmospheric drag measurements

Table 1. Mean position errors (km) due to white noises of accelerometer

<table>
<thead>
<tr>
<th>Altitude (km)</th>
<th>Standard derivation of drag acceleration measurement error $(m/s^2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$10^{-8}$</td>
</tr>
<tr>
<td>200</td>
<td>$2.98 \times 10^3$</td>
</tr>
<tr>
<td>300</td>
<td>$4.13 \times 10^4$</td>
</tr>
<tr>
<td>400</td>
<td>$5.68 \times 10^5$</td>
</tr>
<tr>
<td>500</td>
<td>–</td>
</tr>
<tr>
<td>600</td>
<td>–</td>
</tr>
</tbody>
</table>

Table 2 shows that the position accuracy improves with the decrease of star sensor measurement error. As the standard derivation of star sensor measurement error decreases for one order of mag-

7
Table 2. Mean position errors (km) due to white noises of star sensor

<table>
<thead>
<tr>
<th>Altitude (km)</th>
<th>Standard derivation of star sensor measurement error (second of arc)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>10.1</td>
</tr>
<tr>
<td>200</td>
<td>3.65 × 10^3</td>
</tr>
<tr>
<td>400</td>
<td>5.13 × 10^3</td>
</tr>
<tr>
<td>600</td>
<td>6.58 × 10^3</td>
</tr>
<tr>
<td>1</td>
<td>450</td>
</tr>
<tr>
<td>1</td>
<td>610</td>
</tr>
<tr>
<td>0.1</td>
<td>504</td>
</tr>
<tr>
<td></td>
<td>504</td>
</tr>
<tr>
<td></td>
<td>504</td>
</tr>
<tr>
<td></td>
<td>504</td>
</tr>
</tbody>
</table>

Altitude, the position accuracy increases for one order of magnitude. Moreover, the position accuracy declines slightly with the increase of altitude. This can be explained that in the proposed method, the drag acceleration measurements are finally transformed to spacecraft velocities. The measurement error from star sensor is transformed to the error of velocity direction. In Keplerian theory, when altitude is higher, the velocity direction change will have a greater impact on the position.

Performance for spacecraft navigation

To test the orbit determination method proposed above, a 24h LEO is simulated considering perturbations including earth nonspherical gravity, solar and lunar gravity, solar radiation and atmospheric drag. The initial orbital elements is set as Table 3 shows. The measurements of non-conservative force acceleration are generated from two parts: atmospheric drag and solar radiation pressure. Atmospheric drag acceleration is generated using NRLMSISE00 earth atmosphere model. Solar radiation pressure acceleration is generated by an analytical ephemeris of the Sun and Moon. A white noise with a standard deviation of 1 × 10^{-10} m/s^2 is added to the non-conservative force acceleration signals in order to see the influence of measurement error. The tri-axis accelerometer body frame is set to constantly coincide with the spacecraft body frame and VVLH frame. Attitude measurements of the star sensor are provided in the form of Euler angles, to which white noise with a standard deviation of 1 second of arc is added. An analytical orbit predictor which considers J2, J3 and J4 earth perturbation is adopted in the orbit determination equations because it is fast and accurate enough for LEO.

Table 3. Initial orbital elements of the simulated LEO orbit

<table>
<thead>
<tr>
<th>Orbital element</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>a (km)</td>
<td>6678.136</td>
</tr>
<tr>
<td>e</td>
<td>0.002</td>
</tr>
<tr>
<td>i (°)</td>
<td>50</td>
</tr>
<tr>
<td>Ω (°)</td>
<td>110</td>
</tr>
<tr>
<td>ω (°)</td>
<td>0</td>
</tr>
<tr>
<td>M (°)</td>
<td>5</td>
</tr>
</tbody>
</table>

As shown in Figure 3, the accuracy of the position solutions is on the order of hundreds of meters with a mean error of 625 m and the accuracy of the velocity solutions is on the order of 10^{-1} m/s with a mean error of 0.3483 m/s.

In order to clarify the range of application of the proposed method, simulations are performed
with different altitudes of orbit and different standard derivations of measurement white noise. No white noise is added to the attitude measurements here and the results is shown in Figures 4 and 5. Figure 4 shows that position error increases with the increase of orbit altitude and standard derivation of measurement white noise. Figure 5 shows that velocity error increases with the increase of orbit altitude and standard derivation of measurement white noise mainly. However, when the altitude is too low (about 200km to 270km), the velocity error increases slightly with the decrease of altitude. This anomaly is easy to explain: In this orbit determination method, an approximate orbit predictor which includes J2, J3 and J4 perturbation is adopted. When the orbit altitude is too low, the effect of atmosphere perturbation becomes larger and the accuracy of the orbit predictor reduces slightly. In general, considering the noise density level of the accelerometer nowadays,
the proposed orbit determination method can be applied to the spacecraft which is in an orbit with altitude of 200-550km.

![Figure 4](image4.png)

**Figure 4.** Position error varies with orbit altitude and standard derivation of measurement white noise

![Figure 5](image5.png)

**Figure 5.** Velocity error varies with orbit altitude and standard derivation of measurement white noise

**CONCLUSION**

An autonomous low-Earth orbit determination method based on the atmospheric drag measurement of a tri-axis accelerometer is proposed. A high-precision tri-axis accelerometer is used in order to obtain the atmospheric drag acceleration vector and a star sensor is used for the trans-
formation from the satellite body-fixed coordinate system to the ECI coordinate system. A set of equations is formulated to determined the current position and velocity of the spacecraft using drag measurements of at least four epochs. Specific earth atmosphere model is avoided to be used so that the uncertainty of the model will not affect the accuracy of calculation. The preprocessing of accelerometer measurements is introduced. A set of analytical initial values of position and velocity is provided under the assumption of circular orbit. The effect of measurement noise on the accuracy of orbit determination result is analyzed by conducting several calculations. To test the performance for spacecraft navigation, a high precision orbit is simulated and a series of accelerometer measurements are generated under a white noise environment. The calculation results show good accuracy of position and velocity. The range of application of the proposed method is demonstrated by performing several simulations with different orbit altitudes and different standard derivations of measurement white noise.

REFERENCES